Objectives:
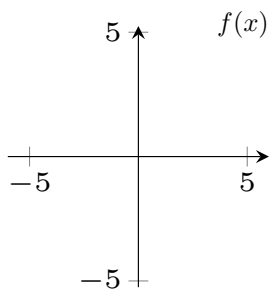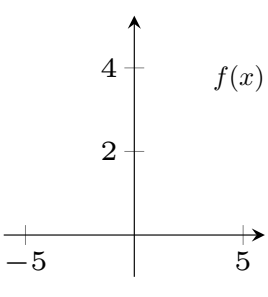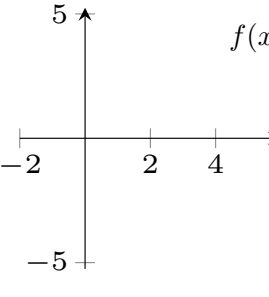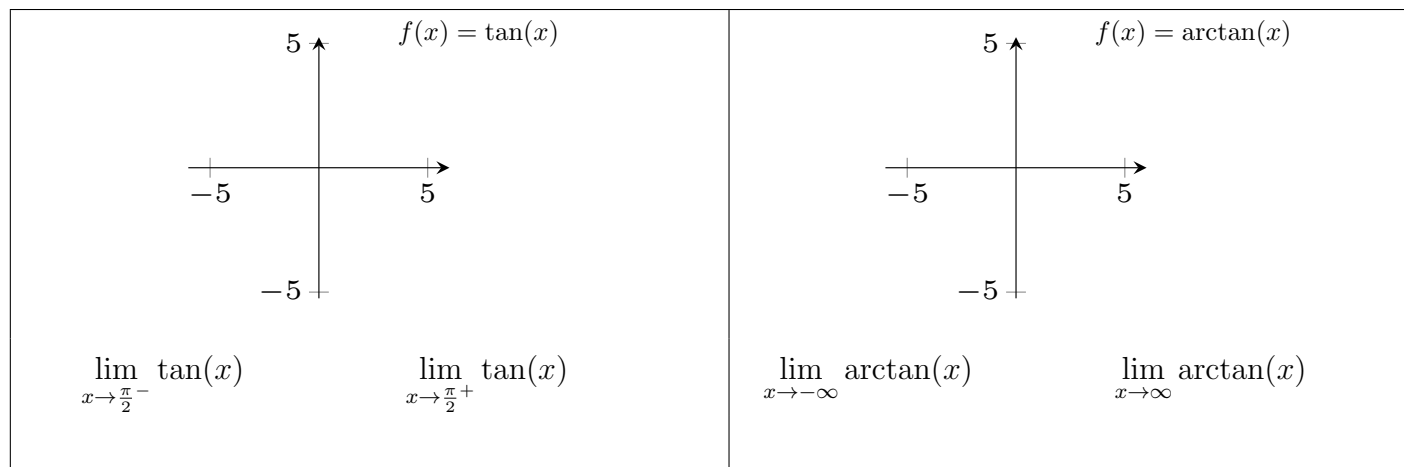
- Find limits where the variable goes to infinity and/or the limit is infinite.

- Find vertical and horizontal asymptotes of a given function.

- Choose and use an appropriate strategy to use with a given indeterminant form.

Limits Involving Infinity Graphically:

- $\displaystyle\lim_{x\to a^+} f(x) = \pm\infty$ and $\displaystyle\lim_{x\to a^-} f(x) = \pm\infty$ represent _____ .

- $\displaystyle\lim_{x\to\infty} f(x) = L$ and $\displaystyle\lim_{x\to-\infty} f(x) = L$ represent _____ .

Common Functions with Limits Involving Infinity:

$$f(x) = \frac{1}{x}$$

$$\lim_{x\to\infty} \frac{1}{x} \qquad\qquad \lim_{x\to 0^+} \frac{1}{x}$$

$$\lim_{x\to-\infty} \frac{1}{x} = \qquad\qquad \lim_{x\to 0^-} \frac{1}{x}$$

$$f(x) = \frac{1}{x^2}$$

$$\lim_{x\to\infty} \frac{1}{x^2} \qquad\qquad \lim_{x\to 0^+} \frac{1}{x^2}$$

$$\lim_{x\to-\infty} \frac{1}{x^2} \qquad\qquad \lim_{x\to 0^-} \frac{1}{x^2}$$

$$f(x) = e^x$$

$$\lim_{x\to\infty} e^x \qquad\qquad \lim_{x\to-\infty} e^x$$

$$f(x) = \ln(x)$$

$$\lim_{x\to\infty} \ln(x) \qquad\qquad \lim_{x\to 0^-} \ln(x)$$

$f(x) = \tan(x)$	$f(x) = \arctan(x)$
$\displaystyle\lim_{x\to\frac{\pi}{2}^-} \tan(x)$ $\displaystyle\lim_{x\to\frac{\pi}{2}^+} \tan(x)$	$\displaystyle\lim_{x\to-\infty} \arctan(x)$ $\displaystyle\lim_{x\to\infty} \arctan(x)$

Using These Common Functions:

Be careful with composite functions! Remember that the direction of the limit of the outside function depends on whether the inside function is increasing or decreasing.

1. $\displaystyle\lim_{x\to\infty} e^{\frac{1}{x}}$

2. $\displaystyle\lim_{x\to 0^+} e^{\frac{1}{x}}$

3. $\displaystyle\lim_{x\to 0^+} \ln(2^x)$

4. $\displaystyle\lim_{x\to\infty} \frac{1}{\ln x}$

5. $\displaystyle\lim_{x\to\infty} \ln\left(\frac{1}{x}\right)$

6. $\displaystyle\lim_{x\to\infty} \sin(\arctan x)$

Indeterminate Forms

Remember that we call the form "$\dfrac{0}{0}$" indeterminate. The forms _____ and _____ are also indeterminate. (We will see even more types of indeterminate forms later on.)

Useful Strategy:

Indeterminate Form Examples:

1. $\lim\limits_{x \to \infty} \dfrac{2x^2 + 3}{x^2 + x}$

2. $\lim\limits_{x \to \infty} \dfrac{3x - 1}{x^2 + 4}$

3. Find horizontal asymptotes of $f(x) = \dfrac{5x^2 + 7}{2x - 4}$

4. $\lim\limits_{x \to \infty} \dfrac{x}{\sqrt{x^2 + 1}}$

5. $\lim\limits_{x \to \infty} \left(\sqrt{x^2 + 1} - x \right)$

6. $\lim\limits_{x \to -\infty} \left(\sqrt{x^2 + 1} - x \right)$